

# Experimental Tests of the Projection Postulate

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## Abstract

Bell [Helv. Phys.Act. 48 1975] described an idealized test of the projection postulate in order to illustrate the ambiguities in the experimental predictions of quantum theory. An approach to the implementation of such tests is outlined here, based on the interesting logical relationship between the projection postulate and the no-superluminal-signaling principle. Gisin's theorem [Helv. Phys. Act. 62 1989] shows that the no-signaling principle implies that any description of projection at the elementary level must be nondeterministic. The lack of determinism means that information cannot be instantiated in a reproducible and transmittable form in isolated microsystems. The implied limitation on information allows one to interpret the no-signaling principle in purely physical terms as a prohibition of superluminal information transmission. This prohibition, in turn, imposes tight constraints on the connection between projection and elementary processes, leading to a specific hypothesis about how projection effects originate. The hypothesis serves as the basis for the experimental proposal.

## 1 Introduction

The projection postulate is the essential link connecting quantum theory to experiment, but it is often regarded as more of an embarrassment than a key physical principle. For example, few people who are active in quantum foundations and quantum information believe that projection is a real physical effect. Two recent surveys of foundational attitudes among researchers found that only 9 per cent and 16 per cent of those questioned believe in objective collapse[1, 2]. The corresponding fractions for belief in physical action-at-a-distance, which is implied by objective collapse, were 12 per cent and 18 per cent. This general view stems from the fact that projection is both nonlocal and nondeterministic, and these properties make it very difficult to see how projection effects could be embedded in a relativistic spacetime structure.

Despite this skepticism, questions about the status of the projection postulate and the reality of the wave function must be addressed by any attempt to understand the fundamental nature of quantum theory. Does the wave function represent a

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genuine physical entity, and can it be truncated by the interactions that constitute measurements? Discussions about foundational issues often ignore the fact that the question of whether such changes occur is, at least in principle, subject to experiment.

The clear inconsistency between the *experimental predictions* of projection versus linear evolution was emphasized by Bell[3]. To evaluate the relationship between wave function collapse and decoherence in the Coleman-Hepp model[4] he constructed observables to demonstrate the persistence of superposition effects to any level of complexity:

“So long as nothing, in principle, forbids the consideration of such arbitrarily complicated observables, it is not permitted to speak of wave packet reduction. While for any given observable one can find a time for which the unwanted interference is as small as you like, for any given time one can find an observable for which it is as big as you do *not* like.”

Aside from any questions of the practicality of testing the projection postulate, the fact that it is testable in principle has important implications for debates about quantum foundations. First, it immediately undercuts the claim that it makes no difference where one draws the line between the measured system and the measuring apparatus. This claim is central to the Copenhagen interpretation[5, 6, 7]. Second, it shows that various interpretations *can be falsified* depending on whether or not they predict (or permit) wave function collapse. No-collapse interpretations such as that of Everett[8], decoherence accounts[9], or pilot wave theories[10, 11, 12] would be falsified by clear evidence of the breakdown of superposition, and collapse proposals such as that of Ghirardi, Rimini, and Weber (GRW)[13] and related hypotheses[14, 15, 16, 17] would be undermined if perfect superposition could be shown to persist to arbitrarily large scales of interaction.

In a number of his later works Bell reiterated his belief that this inconsistency in the experimental predictions of the theory is *the central problem* in quantum foundations[18]:

“The ‘Problem’ then is this: how exactly is the world to be divided... How many electrons, or atoms, or molecules, make an ‘apparatus’?”

The hypothesis that projection is a real physical effect forces one to confront Bell’s question. This is what makes reconsideration of the postulate worthwhile, even if one is strongly inclined to regard it as a temporary patchwork designed to cover our lack of understanding.

The case for taking the projection postulate more seriously is made as follows. In Section 2 Bell’s gedanken experiment is presented in a slightly modified form, and its implications are briefly discussed. Section 3 deals with the problem of reconciling the projection postulate with the relativistic structure of spacetime. I argue there that the perceived difficulty stems from the failure to see that the relativistic description is just as much an “operational” account based on macroscopic observations as

the description of elementary systems in terms of wave functions. Since the incorporation of quantum theory into the relativistic framework depends critically on the no-superluminal-signaling constraint (formalized as the requirement of 'local commutativity'), several key aspects of this principle are studied in Section 4. Gisin's use of the principle to derive several specific conclusions about what happens at the elementary level[19, 20] is contrasted with Maudlin's demonstration[21] that the principle *cannot* be regarded as a prohibition of superluminal information transmission if the concept of 'information' is treated abstractly and applied to arbitrarily small systems. Maudlin's result appears to rule out a clear interpretation of the no-signaling principle in *fundamental physical terms*, and thus calls into question the applicability of the principle to elementary processes. The assumption that projection is a real physical effect is invoked at this point. Using Gisin's result that this implies some lack of determinism in fundamental processes, it is argued that 'information' cannot be instantiated in a reproducible and transmittable form in isolated elementary systems, and that it is only fully realizable at a level of complexity at which projection is complete. This idea is then exploited to suggest how projection arises from the fundamental nondeterministic processes. Section 5 further develops the idea that the limitations on the concept of information apply as much to the definition of reference frames as to the application of the quantum formalism. The additional spacetime structure that is necessary to accommodate the nonlocal and nondeterministic effects that give rise to projection lies below the threshold at which information *and reference frames* can be fully defined. Section 6 uses some of the inferences about how projection arises to show how Bell's gedanken experiment can be brought closer to reality. Section 7 concludes the discussion.

## 2 Bell's Gedanken Experiment

Projection implies the elimination of superposition after a finite number of interactions. Bell showed that this results in differences in some predicted measurement correlations depending on when the projection is assumed to occur. Measurements consist of entangling interactions. The key to testing for superposition effects after entangling interactions occur is to observe the system in a basis that is different from the basis in which the initial interactions take place. For example, one designs an experiment to correlate the x-components of spin of an elementary particle to the states of a target system, and then tests for correlations between the subject particle and the detector system in the y or z-spin basis. This is essentially what Bell proposed in [3]. It is the central idea in the quantum eraser experiments proposed by Scully and his colleagues[22, 23, 24, 25, 26], and it is the basic "trick" in quantum computing[27]. The illustration presented here is a somewhat modified version of Bell's example<sup>1</sup>.

Consider a spin-1/2 particle in a z-up state. If the x-up and x-down components

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<sup>1</sup>Essentially the same example was presented earlier in [28].

are somehow separated and then recombined the z-up state can be recovered if the phases are properly controlled:

$$|z\uparrow\rangle \implies (1/\sqrt{2})(|x\uparrow\rangle + |x\downarrow\rangle) \implies |z\uparrow\rangle. \quad (2.1)$$

Suppose now that we add a "detector" particle in an x-down state and design an interaction so that a subject particle in an x-up state flips the state of the detector from down to up, while a subject particle in an x-down state leaves the detector unchanged. The evolution of the system can be represented schematically as follows:

$$\begin{aligned} |z_0\uparrow\rangle \otimes |x_1\downarrow\rangle &\implies (1/\sqrt{2})(|x_0\uparrow\rangle + |x_0\downarrow\rangle) \otimes |x_1\downarrow\rangle \\ &\implies (1/\sqrt{2})(|x_0\uparrow\rangle|x_1\uparrow\rangle + |x_0\downarrow\rangle|x_1\downarrow\rangle) = (1/\sqrt{2})(|z_0\uparrow\rangle|z_1\uparrow\rangle + |z_0\downarrow\rangle|z_1\downarrow\rangle). \end{aligned} \quad (2.2)$$

The  $|z_0\uparrow\rangle$  state can no longer be detected in 100 per cent of the cases, but superposition effects are still exhibited through the perfect correlations between  $|z_0\uparrow\rangle$  and  $|z_1\uparrow\rangle$ , and between  $|z_0\downarrow\rangle$  and  $|z_1\downarrow\rangle$ .

This general schema can be applied to quantum eraser experiments. The two different representations of the final state,  $(1/\sqrt{2})(|x_0\uparrow\rangle|x_1\uparrow\rangle + |x_0\downarrow\rangle|x_1\downarrow\rangle)$ , and  $(1/\sqrt{2})(|z_0\uparrow\rangle|z_1\uparrow\rangle + |z_0\downarrow\rangle|z_1\downarrow\rangle)$ , correspond to complementary observables. The x-representation can be thought of as containing the "which-path" information, while the z-representation can be regarded as exhibiting "interference" effects.

The important point here is that we believe that the "interference" terms can still be seen because the elementary interaction between  $|x_0\uparrow\rangle$  and the  $|x_1\rangle$  "detector" system did not result in a (complete) projection to either  $(|x_0\uparrow\rangle|x_1\uparrow\rangle)$  or  $(|x_0\downarrow\rangle|x_1\downarrow\rangle)$ .

To see this explicitly, one can expand the two x-branches separately in terms of the z-spin basis.

$$\begin{aligned} (1/\sqrt{2})|x_0\uparrow\rangle|x_1\uparrow\rangle &= (1/\sqrt{2})^3(|z_0\uparrow\rangle + |z_0\downarrow\rangle) \otimes (|z_1\uparrow\rangle + |z_1\downarrow\rangle) \\ &= (1/\sqrt{2})^3(|z_0\uparrow\rangle|z_1\uparrow\rangle + |z_0\downarrow\rangle|z_1\downarrow\rangle + |z_0\uparrow\rangle|z_1\downarrow\rangle + |z_0\downarrow\rangle|z_1\uparrow\rangle) \end{aligned} \quad (2.3)$$

and

$$\begin{aligned} (1/\sqrt{2})|x_0\downarrow\rangle|x_1\downarrow\rangle &= (1/\sqrt{2})^3(|z_0\uparrow\rangle - |z_0\downarrow\rangle) \otimes (|z_1\uparrow\rangle - |z_1\downarrow\rangle) \\ &= (1/\sqrt{2})^3(|z_0\uparrow\rangle|z_1\uparrow\rangle + |z_0\downarrow\rangle|z_1\downarrow\rangle - |z_0\uparrow\rangle|z_1\downarrow\rangle - |z_0\downarrow\rangle|z_1\uparrow\rangle) \end{aligned} \quad (2.4)$$

The correlations in 2.2 result from the cancellation of the up-down cross terms,  $(|z_0\uparrow\rangle|z_1\downarrow\rangle)$  and  $(|z_0\downarrow\rangle|z_1\uparrow\rangle)$ , that occurs when 2.3 and 2.4 are added. If we were to regard the interaction between  $|x_0\uparrow\rangle$  and the  $|x_1\rangle$  system as a "measurement", we would expect projection to one of the two x-branches with the other branch being eliminated. There would be no cancellation of cross terms and no "interference", i.e., the z-spin correlations would completely disappear.

What Bell showed is that the differences in the experimental predictions of continued superposition as opposed to projection are *not* removed by adding any finite number of interactions.

Suppose that the single detector particle,  $|x_1\rangle$ , is replaced by  $N$  detectors,  $|x_{d1}\rangle$ ,  $|x_{d2}\rangle$ , ...,  $|x_{dN}\rangle$ . The subject particle is still labeled,  $|x_0\rangle$ , and the interactions are again designed so that an x-up subject state flips the detector particles from down to up, and an x-down state leaves them unchanged. The initial preparation state:

$$(1/\sqrt{2})(|x_0\uparrow\rangle + |x_0\downarrow\rangle) \otimes (|x_{d1}\downarrow\rangle|x_{d2}\downarrow\rangle\cdots|x_{dN}\downarrow\rangle)$$

evolves to:

$$(1/\sqrt{2})(|x_0\uparrow\rangle \otimes (|x_{d1}\uparrow\rangle|x_{d2}\uparrow\rangle\cdots|x_{dN}\uparrow\rangle) + (1/\sqrt{2})(|x_0\downarrow\rangle) \otimes (|x_{d1}\downarrow\rangle|x_{d2}\downarrow\rangle\cdots|x_{dN}\downarrow\rangle).$$

Now expand the x-up and x-down branches in the z-spin basis:

$$\begin{aligned} & (1/\sqrt{2})|x_0\uparrow\rangle|x_{d1}\uparrow\rangle\cdots|x_{dN}\uparrow\rangle \\ = & (1/\sqrt{2})^{N+2}[(|z_0\uparrow\rangle + |z_0\downarrow\rangle) \otimes (|z_{d1}\uparrow\rangle + |z_{d1}\downarrow\rangle)\cdots(|z_{dN}\uparrow\rangle + |z_{dN}\downarrow\rangle)] \end{aligned} \quad (2.5)$$

and

$$\begin{aligned} & (1/\sqrt{2})|x_0\downarrow\rangle(|x_{d1}\downarrow\rangle\cdots|x_{dN}\downarrow\rangle) \\ = & (1/\sqrt{2})^{N+2}[(|z_0\uparrow\rangle - |z_0\downarrow\rangle) \otimes (|z_{d1}\uparrow\rangle - |z_{d1}\downarrow\rangle)\cdots(|z_{dN}\uparrow\rangle - |z_{dN}\downarrow\rangle)]. \end{aligned} \quad (2.6)$$

If 2.5 and 2.6 are expressed as sums of products of the form  $|z_0\rangle|z_{d1}\rangle|z_{d2}\rangle\cdots|z_{dN}\rangle$ , the terms in the two equations would be identical, but in 2.6 half of them would have plus signs and half would have minus signs. This makes it convenient to rewrite the two expressions as:

$$\begin{aligned} & (1/\sqrt{2})^3[|z_0\uparrow\rangle|z_d\downarrow_{EVEN}\rangle + |z_0\downarrow\rangle|z_d\downarrow_{ODD}\rangle + |z_0\uparrow\rangle|z_d\downarrow_{ODD}\rangle + |z_0\downarrow\rangle|z_d\downarrow_{EVEN}\rangle], \\ & (1/\sqrt{2})^3[|z_0\uparrow\rangle|z_d\downarrow_{EVEN}\rangle + |z_0\downarrow\rangle|z_d\downarrow_{ODD}\rangle - |z_0\uparrow\rangle|z_d\downarrow_{ODD}\rangle - |z_0\downarrow\rangle|z_d\downarrow_{EVEN}\rangle], \end{aligned}$$

where the following definitions have been used:

$$\begin{aligned} |z_d\downarrow_{EVEN}\rangle & \equiv (1/\sqrt{2})^{N-1} \sum_{even|z_{di}\downarrow} (\prod_i |z_{di}\rangle), \\ |z_d\downarrow_{ODD}\rangle & \equiv (1/\sqrt{2})^{N-1} \sum_{odd|z_{di}\downarrow} (\prod_i |z_{di}\rangle), \end{aligned}$$

where  $i$  ranges from 1 to  $N$ . In other words  $|z_d\downarrow_{EVEN}\rangle$  is the normalized sum of all product detector states with an even number of  $|z_{di}\downarrow\rangle$  occurrences, and  $|z_d\downarrow_{ODD}\rangle$  is the corresponding sum with an odd number of  $|z_{di}\downarrow\rangle$  occurrences.

If no projection effects have occurred after the  $N$  correlating interactions in the x-spin basis, then the two branches represented by 2.5 and 2.6 can be superposed. This results in cancellation of the  $|z_0\uparrow\rangle|z_d\downarrow_{ODD}\rangle$  terms and the  $|z_0\downarrow\rangle|z_d\downarrow_{EVEN}\rangle$  terms. So the signature of continued superposition is now exhibited as a perfect correlation between up results of z-spin measurements on the subject particle, and an *even* number of down results from z-spin measurements on the detector particles, and a corresponding correlation between down results of z-spin measurements on the subject particle, and an *odd* number of down results from z-spin measurements on the detector particles.

So wave function collapse *is observable* in principle. An experiment showing a statistically significant number of coincidences of z-up results on a subject particle and a  $|z_d\downarrow_{ODD}\rangle$  state would be strong evidence of the breakdown of superposition.

Obviously, as the number of interactions,  $N$ , increases it becomes more difficult to track the z-spin states of all of the detector particles. These are the typical effects of decoherence. The challenge in detecting the correlations is similar to that of

constructing a quantum computer. One must carefully design the interactions that generate the entangled states, and then accurately track the evolution of all of the particles involved.

The technical challenges of such an experiment would be very formidable, but one of the biggest reasons that almost nobody considers actually doing this sort of test is that the nonlocal and nondeterministic nature of projection makes it difficult to reconcile with the standard relativistic picture of spacetime. This issue is addressed in the next three sections which reexamine the relationship between relativity and projection.

### 3 Relativity and Nonlocal Effects

Among those who try to resolve the apparent conflicts between quantum theory and relativity, the dominant tendency has been to place the full burden of reconciliation on quantum theory. For example, the surveys cited in the opening paragraph included another question which concerned preferred interpretations of the quantum state. The first three options were: (a) epistemic/informational, (b) ontic, (c) a mix of epistemic and ontic. This is obviously an issue worth exploring, but the point is that this kind of question is very rarely raised about the status of the relativistic structure of spacetime<sup>2</sup>.

In one respect this is rather surprising. In his original (1905) paper on relativity[29] Einstein emphasized the need to define reference frames in terms of physical relationships among things like rigid rods and clocks. Consider the following excerpts:

”... since the assertions of any such theory have to do with the relationships between rigid bodies (systems of co-ordinates), clocks, and electromagnetic processes.”

”Now we must bear carefully in mind that a mathematical description of this kind has no physical meaning unless we are quite clear as to what we understand by time.”

Einstein’s reason for emphasizing the dependence of such definitions on references to physical objects and processes was his recognition that the laws of physics seriously constrain the ways in which such definitions can be formulated. The constraints that Einstein had in mind stemmed from the fact that the speed of light is a constant, independent of reference frame. But there are other laws of nature, unknown to physicists at the time, that are equally important in guaranteeing the consistency of the relativistic description of physical processes with experimental results. These include various implications of quantum theory such as the Born probability rule[30] and the no-cloning theorem[31].

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<sup>2</sup>Except in pilot wave theory.

Somewhat later (1921) Einstein anticipated the possibility that macroscopically based metric definitions might not apply on the microphysical level[32].

”It is true that this proposed physical interpretation of geometry breaks down when applied ...to spaces of submolecular ...magnitude. ... It might ...turn out that this extrapolation has no better warrant than the extrapolation of the concept of temperature to parts of a body of molecular... magnitude.”

We should not be misled by the fact that, when quantum theory was more fully developed, physicists were able to describe the interactions of elementary particles within a relativistic framework. This framework is *not* a straightforward extension of the classical relativistic spacetime description. It was only made possible by placing a still unexplained discontinuity between macroscopic and microphysical domains, and by introducing a decidedly nonclassical and nonintuitive account of the particles and processes that are used to infer the properties of the spacetime structure. The ontological status of that spacetime structure is at least as open to question as the status of those particles and processes.

The analysis of Bell[33, 34] and its empirical confirmation by Aspect[35, 36] clearly show that there are real nonlocal effects. The difficulties in accounting for these effects in a way that conforms to all of our intuitions about relativity should lead us to question our assumptions about the fundamental nature of the relativistic framework - not to deny the clear evidence of experiment.

The argument of the previous section shows that there are ambiguities (or inconsistencies) in the experimental predictions of quantum theory depending on where one places the cut between the macro and micro domains. These discrepancies are not mere pedantic quibbles, even if they are very difficult to detect. To deal with these discrepancies we must take seriously the possibility that there is a different and deeper spacetime structure underlying the relativistic description of observed phenomena.<sup>3</sup>

We cannot properly assess the role of the relativistic description of spacetime within contemporary theory until we more fully understand the connection between elementary processes and measurement outcomes. To see this, consider how quantum theory maintains consistency with that description. Relativity implies that there can be no differences in any observable effects when the sequencing of spacelike-separated events is interchanged. So, if spacelike-separated measurements are made on an entangled system, their temporal ordering must not affect the outcome probabilities. For arbitrary measurement results,  $a$  and  $b$ , the probability of  $b$ , given  $a$ , must be equal to the probability of  $a$ , given  $b$ :  $P(b|a) = P(a|b)$ . This symmetry in the measurement probabilities is maintained within the vector space representation of quantum states by requiring the relative probability to be a simple function of the projection (in the ordinary vector space sense) of the unit vector,  $\vec{a}$  onto  $\vec{b}$ , which is, of course, equal to

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<sup>3</sup>This is one of the central themes of pilot wave theories[12, 37, 38].

the projection of  $\vec{b}$  onto  $\vec{a}$ .<sup>4</sup> Note that the same projection relationship is responsible for the quantum correlations in Bell-EPR states[33, 34, 39] that exceed any that can be explained in terms of locally propagating influences. In other words, the consistency of the relativistic description with observations is preserved by the same vector space relationship and measurement postulate that explain the distinctive nonlocal correlations that appear to be so difficult to reconcile with relativity. Until we can explain how the measurement postulates are related to fundamental processes, we do not know what relativity is really telling us about the underlying spacetime structure.

To investigate how measurement outcomes are connected to elementary processes we must design experiments to determine what happens at the quantum/classical boundary. To understand how projection fits into the overall theoretical framework we need to examine more carefully the logical structure of contemporary theory. The key to insuring that quantum theory respects relativistic symmetries is the postulate of 'local commutativity', the assumption that spacelike-separated field operators commute. In quantum field theory texts, this postulate is often described as a 'causality' requirement. This has often led to the unfortunate confusion of local commutativity with the concept of local causality, the principle that all physical processes propagate with a speed less than or equal to that of light. In fact, local commutativity is a much less restrictive requirement than local causality. As noted by Bell[40] and others[21, 41] local commutativity is essentially equivalent to what is called 'no-superluminal-signaling' among researchers in quantum foundations and quantum information. In the next section, I shall argue that the no-signaling principle provides a valuable clue as to how nonlocal quantum effects arise from fundamental processes.

## 4 No-Signaling, Projection, and Elementary Interactions

The no-signaling principle is generally regarded as essential to the relativistic description of spacetime, and it can also be used to derive many of the key properties of quantum theory. For example, it implies Gleason's assumption of noncontextuality[42], thus yielding the Born probability rule. Gisin used it to show that any deterministic quantum evolution must be linear, and that any description of projection at the elementary level must be nondeterministic[19, 20]. Other important consequences have been demonstrated by a number of authors.[43, 44, 45, 46, 47, 48]. The general reasons that the principle proves so useful have been studied by Svetlichny[49, 50]. He points out that nonlocal effects lead to violations of no-signaling unless they are very tightly constrained.

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<sup>4</sup>The specific dependence on the square of the projection length follows from some elementary considerations involving probability and analyticity. This is a consequence of Gleason's theorem.[42]



The goal here is to use the principle to draw specific inferences about how projection can be explained in terms of elementary processes. The basic idea is illustrated in the following heuristic argument.

Suppose that the wave function of a particle has bifurcated into two principal components, and that a measurement apparatus has been set up to determine whether it is in one of the branches. A negative outcome indicates that the wave function has collapsed, and that the particle is localized in the other branch. This measurement clearly reveals information about the state of the (spacelike-separated) particle. If this collapse were not induced by the physical processes that constitute the measurement, then the measurement would be *revealing information about a random event that occurred at spacelike separation*. The prohibition of superluminal information transmission would be violated. Therefore projection must be induced by some physical processes that are essential to measurement. At the most basic level, measurements establish correlations among the states of elementary particles. So, the elementary interactions that generate the correlations are the source of projection effects.

This heuristic argument is obviously based on an understanding of the no-signaling principle as a complete prohibition of *any superluminal information transmission*. This is somewhat more general than the way in which the no-signaling principle is often understood. The more limited interpretation of the principle carries an implicit reference to communication between intelligent observers. It was this more limited formulation that was disparaged by Bell[40]. He argued that the concept of 'signaling' was both vague and subjective since discussions of it typically make reference to humans, or at least to our high-level artefacts. Because of this, he believed that it was very unlikely to serve as a bridge between macroscopic experimental results and elementary interactions.

In support of the more general understanding of the principle proposed here, one can cite two points. First, as stated at the close of the previous section, the no-signaling principle is viewed as an acceptable 'translation' of local commutativity into ordinary language. Since local commutativity is an essential feature of relativistic formulations of quantum theory<sup>5</sup>, it ought to be understood in terms of objective *physical* properties, independent of any reference to intelligent observers. Second, as mentioned above, the principle has already been employed by Gisin[19] to infer a specific constraint on the connection between projection and elementary processes, viz., that it must be nondeterministic.

Before we can apply this more general understanding of no-signaling, we need to show that the concept of information as a property of *physical* systems has a clear meaning that is consistent with this interpretation of the principle. This requires some care since Maudlin[21] has shown that if isolated elementary particles are regarded as instantiating information, then that information *is* transmitted superluminally. So, if the no-signaling principle is to place meaningful constraints on fundamental processes,

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<sup>5</sup>One can regard it either as an assumption, or as a consequence of Lorentz invariance, but in either case it is a necessary property of the theory.

we need to develop an alternate notion of information, one that excludes the type of situation examined by Maudlin, but still has a clear physical interpretation.

The key to developing such a notion is the recognition that, *if* projection is a real physical effect, then some fundamental processes are not deterministic, and this implies that, in general, the state of an isolated elementary particle cannot be faithfully transferred through a chain of correlating interactions with other particles (as shown in the no-cloning theorem[31]). The physical processes that result in projection modify quantum states in ways that are not completely traceable, and thus make it impossible, in principle, to extract specific information about the original state of the particle.

The concept of information ordinarily includes the notions of reproducibility and transmittability. So let us consider how information that possesses these properties is generated when elementary processes are not completely deterministic. In particular, we are concerned with information about quantum states.

Consider a large collection of independent particles in identical, but unknown spin states. One can acquire the information about which state the particles are in by measuring the spin along axes at a variety of angles that cover a sphere. The distribution of outcomes will indicate the original state with a precision dependent on the number of measurements made. This statistical inference is only legitimate because the measurement outcomes are independent. So a measurement on one particle must not affect the state of any other particles. In other words, it is essential that the original combined state of the collection of particles is factorizable.

Let us compare the situation just described to what happens in a measurement on a single particle in an unknown spin state, using an idealized arrangement similar to that of section 2. The experimenter chooses an axis, which can be labeled 'x', along which to measure. The  $|x_0 \uparrow\rangle$  and  $|x_0 \downarrow\rangle$  states are separated, and the x-up branch undergoes an interaction with a set of detector particles. If no projection has occurred after  $N$  correlating interactions the state can be represented as:

$$(\alpha)(|x_0 \uparrow\rangle \otimes (|x_{d1} \uparrow\rangle |x_{d2} \uparrow\rangle \dots |x_{dN} \uparrow\rangle)) + (\beta)(|x_0 \downarrow\rangle \otimes (|x_{d1} \downarrow\rangle |x_{d2} \downarrow\rangle \dots |x_{dN} \downarrow\rangle)),$$

where  $\alpha$  and  $\beta$  are unknown. Even if  $N$  is quite large, as long as the system remains in a superposition of  $|x_0 \uparrow\rangle$  and  $|x_0 \downarrow\rangle$  states, there is no way to take a statistical sample of the states of the detector particles since they are entangled. Any complete x-state measurement on one of them would collapse the entire system to either the  $|x_0 \uparrow\rangle$  or  $|x_0 \downarrow\rangle$  branch. Information about the relative magnitudes of  $\alpha$  and  $\beta$  would be lost. It is only at the stage when projection occurs that the system is transformed back into a factorizable state, and reproducible, transmittable information about the *new* state becomes available.<sup>6</sup>

So, based on the assumption that projection is a real effect, we have a solid *physical*

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<sup>6</sup>One could view the generation of a large number of *independent* correlated particle states by the transformation from an entangled state to a factorizable state as the "irreversible act of amplification" discussed by Bohr[51], but it is important to emphasize that it is information that is being created through projection - not 'physical reality'.

characterization of the conditions for a system to instantiate information about the quantum state of a particle. Given the nondeterministic character of projection, such information exists when the state of the subject particle is represented, *independently*, in the states of a sufficient number of other physical systems. The use of the term 'sufficient number' should not be seen as introducing another form of vagueness into the discussion. The concept of information described here is analogous to the concept of temperature in statistical mechanics. Both of these concepts can be applied to mesoscopic or macroscopic systems. The fact that the requisite size of these systems is usually not precisely specified does not affect the objectivity of these notions.

To emphasize the objectivity of this concept, note that it is possible to develop a well-defined statistical measure of when it happens (even though one cannot determine in individual cases precisely at what stage projection occurs). Let us refer again to the illustration from section 2.

After  $N$  interactions in the x-spin basis the  $|x_0 \uparrow\rangle$  and  $|x_0 \downarrow\rangle$  branches are represented in the z-spin basis as:

$$(1/\sqrt{2})^3[|z_0 \uparrow\rangle|z_d \downarrow_{EVEN}\rangle + |z_0 \downarrow\rangle|z_d \downarrow_{ODD}\rangle + |z_0 \uparrow\rangle|z_d \downarrow_{ODD}\rangle + |z_0 \downarrow\rangle|z_d \downarrow_{EVEN}\rangle], \text{ and } (1/\sqrt{2})^3[|z_0 \uparrow\rangle|z_d \downarrow_{EVEN}\rangle + |z_0 \downarrow\rangle|z_d \downarrow_{ODD}\rangle - |z_0 \uparrow\rangle|z_d \downarrow_{ODD}\rangle - |z_0 \downarrow\rangle|z_d \downarrow_{EVEN}\rangle].$$

If perfect superposition persists to this point then the overall state is:

$$(1/\sqrt{2})(|z_0 \uparrow\rangle|z_d \downarrow_{EVEN}\rangle + |z_0 \downarrow\rangle|z_d \downarrow_{ODD}\rangle). \text{ So an unambiguous observation of a } |z_0 \uparrow\rangle \text{ state correlated with a } |z_d \downarrow_{ODD}\rangle \text{ state would indicate that projection to either the } |x_0 \uparrow\rangle \text{ or } |x_0 \downarrow\rangle \text{ branch had occurred prior to the z-state measurement.}$$

To determine the typical scale at which projection occurs (i.e., the scale at which 'information' becomes definable) one would run this type of experiment a large number of times,  $M$ , and define a measure of superposition, analogous to a correlation coefficient,  $r$ , as follows. Let  $q_i = +1$  for observations that are consistent with continued superposition,  $|z_0 \uparrow\rangle|z_d \downarrow_{EVEN}\rangle$  and  $|z_0 \downarrow\rangle|z_d \downarrow_{ODD}\rangle$ , and let  $q_i = -1$  for observations that indicate collapse,  $|z_0 \uparrow\rangle|z_d \downarrow_{ODD}\rangle$  and  $|z_0 \downarrow\rangle|z_d \downarrow_{EVEN}\rangle$ . Define  $r \equiv (1/M) \sum_i q_i$ .<sup>7</sup> Then the quantity,  $1 - r$ , indicates the fraction of cases in which projection has occurred for any given number of correlating interactions,  $N$ . This particular measure applies to this rather simple, idealized arrangement, but analogous measures could be defined for any system of interacting particles that is sufficiently well specified.

It is important to emphasize that although the example described above involves a very idealized and highly organized system, the intervention of intelligent observers is *not* essential to the applicability of the concept. Information, in the sense intended here, can be instantiated in naturally occurring systems. Perhaps, the most common example is the generation of information about the position of a particle. If the wave function of a particle separates into different branches or simply diffuses over a large enough region, it will eventually interact with systems of particles large enough to relocalize it (either within the region of interaction or outside it). Other natural "information processing devices" might include biological systems. The re-

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<sup>7</sup>Note that the outcomes that are consistent with superposition would also occur in one half of the cases in which collapse has occurred.

cent demonstration that superposition plays a key role in the efficient capture and distribution of energy in algae[52], together with the fact that organisms are capable of reproducing various types of information, suggests that basic metabolic processes are capable of completely reducing the wave function.

As a simple illustration of the fact that the sort of physical information that has been outlined here cannot be transmitted by nonlocal quantum effects consider a pair of elementary particles in a singlet state. If the particles are labeled  $a$  and  $b$ , the state can be represented as:  $(1/\sqrt{2})(|x_a \uparrow\rangle|x_b \downarrow\rangle - |x_a \downarrow\rangle|x_b \uparrow\rangle)$ . After the particles are separated, an x-spin measurement on  $a$  projects it into an  $|x \uparrow\rangle$  or  $|x \downarrow\rangle$  state, and  $b$  into the complementary state. Information about the state of  $a$  is generated in the system of particles that interact with  $a$ . That information is reproducible and transmittable to other systems. The system of particles contains only contingent information about the state of  $b$ , since the information resulting from the set of interactions undergone by  $a$  cannot rule out the possibility that  $b$  has interacted in some alternate basis. If  $b$  does not interact in some organized fashion, then there is no reproducible, transmittable information in  $b$ , or in its immediate vicinity, since the state that would result from any such interactions is uncertain (until the information from the  $a$  system is transmitted to the region of  $b$ ).

So, given the assumption that projection is a real physical effect, information of the relevant sort is generated by projection, which transforms entangled states into factorizable ones, creating multiple representations of the state of the "measured" system. This notion of information enables us to interpret the no-signaling principle (i.e., the prohibition of superluminal information transmission) as an objective, physical principle, without any essential reference to intelligent observers. Furthermore, it reinforces the conclusion of the heuristic argument presented above - that projection effects are induced by the elementary correlating interactions of the sort that constitute measurements.

This hypothesis can be refined and extended. Note first that simple verification measurements, in which the measured system is already in an eigenstate of the measured observable, do not result in projection. The problematic cases are those in which measurement interactions *nonlocally entangle* the states of the subject and detector systems. So let us hypothesize that nonlocal, nondeterministic projection effects are induced by elementary interactions that create entanglement relations between particles that are spacelike-separated.

The nature of these effects can be discerned by considering what happens in measurements at the elementary level. The particles making up the detector either interact or fail to interact with the subject particle<sup>8</sup>, indicating the subject particle's presence or absence. The wave function of the particle changes, reflecting the transfer of amplitude either into or out of the interacting branch. This suggests that

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<sup>8</sup>The interactions can be indirect through a chain of mediating particles.

elementary interactions that generate entanglement also involve a transfer of amplitude either into or out of the interacting component of the wave function.<sup>9</sup>

The binary decomposition of the wave function into interacting and noninteracting branches enables us to view the projection of the state vector as similar to a random walk. The idealized situation described in Section 2 provides a simple illustration of the idea. In that example, after  $N$  correlating interactions the state was represented as:

$$(\alpha)(|x_0 \uparrow\rangle \otimes (|x_{d1} \uparrow\rangle |x_{d2} \uparrow\rangle \dots |x_{dN} \uparrow\rangle)) + (\beta)(|x_0 \downarrow\rangle \otimes (|x_{d1} \downarrow\rangle |x_{d2} \downarrow\rangle \dots |x_{dN} \downarrow\rangle)).$$

The coefficients of the two branches given in Section 2,  $(1/\sqrt{2})$ , have been replaced here by  $\alpha$  and  $\beta$  because they are now being considered as dynamic entities which change in response to the interactions (assumed to be taking place in the  $|x_i \uparrow\rangle$  branch). So, for example, if  $|x_{dN} \uparrow\rangle$  interacts with the  $|x_{dN+1}\rangle$  particle, then  $\alpha$  and  $\beta$  will change to  $\alpha'$  and  $\beta'$ .

Such amplitude transfers must reproduce the Born rule exactly at the macroscopic level, in order to prevent superluminal signaling and preserve relativity. The most straightforward way to do this is to assume that each elementary interaction carries an equal chance to increase or decrease the amplitude of the interacting component of the wave function. Since the probability of an outcome is the absolute square of the amplitude, the amount of the change must be scaled so that the potential increase in squared amplitude equals the potential decrease. In terms of the coefficients above, this implies that  $\alpha'\alpha^* = \alpha\alpha^* \pm \delta$ , and  $\beta'\beta^* = \beta\beta^* \mp \delta$ , where  $\delta$  is some small increment that cannot exceed either  $\alpha\alpha^*$  or  $\beta\beta^*$ .

This type of random walk process was outlined in an earlier work[28]. The fact that the Schrödinger equation is linear with respect to the amplitudes implies that the nonlocal transfers do not result in violations of the no-signaling principle at the observable level. To reconcile these nonlocal and nondeterministic amplitude transfers with the usual relativistic description of spacetime, it is necessary to deal with some issues that arise when a single entangled system undergoes spacelike-separated interactions. These issues are exemplified by EPR-Bell type experiments. In the idealized example described above the sequence of steps in the random walk is clearly determined, and the collapse process is straightforward. However, the standard relativistic temporal ordering does not provide a way to sequence interactions that make up spacelike-separated measurements. As stated in Section 3, at the observable level one can regard either of the two measurements as occurring first. The issue of sequencing is addressed in the next section.

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<sup>9</sup>This assumes that the relevant system has both interacting and noninteracting components. In entanglement-generating interactions such as parametric down conversion, there is no noninteracting component to be involved in any amplitude transfer.

## 5 The Sequencing of Nonlocal Effects and the Evolution of Projection

To have a well-defined projection process and insure definite measurement outcomes, it is essential that there exists some sort of (unobservable) sequence of spacelike-separated interactions involving the same entangled system. This is particularly true because the individual steps are nondeterministic, and this implies that the changes induced in the system are not reversible.[21, 43, 45, 46]

To illustrate the issue, let us first consider a simple situation in which there is no problem with sequencing. Suppose that  $\psi$  and  $\phi$  represent elementary systems, and that  $\psi$  consists of two spatially separated branches,  $\psi_1$ , and  $\psi_2$ , with amplitudes  $\alpha$  and  $\beta$ . The subscripts, 1 and 2, correspond to interacting and noninteracting components, respectively. The state of  $\phi$  prior to any interaction can be labeled  $\phi_0$ , so the state of the combined system prior to interaction is  $(\alpha\psi_1 + \beta\psi_2) \otimes \phi_0$ . Given an appropriate, short duration interaction between  $\psi_1$  and  $\phi_0$ , the initial product state will evolve to the entangled state:  $(\alpha\psi_1 \otimes \phi_1) + (\beta\psi_2 \otimes \phi_0)$ , according to the Schrödinger equation. As described above, the proposed projection hypothesis is that, in parallel with the ordinary Schrödinger evolution, there is also a change in  $\alpha$  and  $\beta$  to  $\alpha'$  and  $\beta'$  with  $\alpha'\alpha^* = \alpha\alpha^* \pm \delta$ , and  $\beta'\beta^* = \beta\beta^* \mp \delta$ .

Problems with sequencing can occur if we introduce a third elementary system  $\chi$  which could interact with  $\psi_2$ . The Schrödinger evolution, would then take the system from  $(\alpha\psi_1 + \beta\psi_2) \otimes \phi_0 \otimes \chi_0$  to  $(\alpha\psi_1 \otimes \phi_1 \otimes \chi_0) + (\beta\psi_2 \otimes \phi_0 \otimes \chi_2)$ . Since both branches of the wave function undergo interactions (presumably at spacelike separation), without some supplementary sequencing there is no noninteracting branch of the wave function to allow for amplitude shifts. If these interactions occur in isolation, one can simply assume that there is no amplitude shift, and that the system continues to evolve strictly according to the Schrödinger equation. However, if the interactions are parts of spacelike-separated measurements of the position of  $\psi$  (or some correlated property), then we expect that one measurement will have a positive outcome, and the other will have a negative outcome.

What is required is some additional ordering parameter that can supplement the standard partial ordering implied by the timelike relations of relativistic spacetime. It is not necessary to have a total ordering of events in spacetime, or even of all of the elementary interactions that a particular entangled system might undergo. The additional structure need only insure that, generically, enough of the interactions that constitute one of a pair (or set) of measurements are sequenced at a stage different from the interactions in the spacelike-separated process.

The kind of ordering that is required can be provided by defining a global sequencing parameter,  $s$ , that is associated with a *randomly evolving* spacelike hypersurface, which will be labeled  $\sigma(s)$ . By allowing  $\sigma(s)$  to evolve forward in a random fashion one can insure that (for the most part) spacelike-separated interactions involving the same entangled system are sequenced at different values of  $s$ . It also maintains

as much consistency as possible with the spirit of relativity by not assuming any preferred reference frame.<sup>10</sup>

With most spacelike-separated interactions being sequenced at different values of  $s$ , there is an unambiguous decomposition of the wave function into interacting and noninteracting branches at each stage of the collapse process. In those cases in which interactions in spacelike-separated regions occur at the same value of  $s$ , the decomposition is trivial, and there is no change in the amplitudes,  $\alpha$  and  $\beta$ . Since real measurements consist of an enormous number of interactions, this possibility does not preclude a definite outcome.<sup>11</sup>

Under this hypothesis, the state vector evolves randomly with respect to  $s$ . At any given stage, the division of the wave function into interacting and noninteracting branches defines a binary decomposition of the Hilbert space of the system into two orthogonal subspaces.<sup>12</sup> When the state vector reaches one of the two subspaces, the distinction between interacting and noninteracting components collapses, and the decomposition becomes trivial. The supplementary evolution ceases until some new binary decomposition is defined by subsequent entangling interactions.

Since this approach assumes that there is an evolving spacelike hypersurface,  $\sigma(s)$ , that determines the actual sequencing of the nonlocal effects, the description of the evolution at this level is not Lorentz covariant. The sequencing remains unobservable, in principle, because the amplitude shifts are nondeterministic. The observed outcome of the projection process is consistent with *any* sequence of spacelike-separated interactions. Any attempt to "watch" the collapse process necessarily involves additional interactions on an entangled system that are subject to the same lack of determinism and unobservable sequencing.

The fundamentally probabilistic nature of the amplitude transfers precludes both a stable physical representation of information *and* the definition of reference frames at the most elementary level. The application of these concepts requires a well-defined network of physical relations, and such a network can only exist at a scale at which probabilistic fluctuations are small compared to relevant parameters. The prohibition of superluminal information transmission and the relativistic description of spacetime emerge together at the level at which reproducible, transmittable information can be

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<sup>10</sup>The sequencing parameter,  $s$ , is similar in many respects to a preferred time coordinate such as occurs in de Broglie-Bohm theory although it does not coincide with the time in any single inertial reference frame. At each point,  $x$ , on  $\sigma(s)$  one can define a proper time derivative,  $d\tau(x)/ds$ , but, in general,  $d\tau(x)/ds \neq d\tau(x')/ds$  for  $x \neq x'$ . If  $d\tau(x)/ds > 0$  at every point,  $x$ , then the succession of hypersurfaces will constitute a foliation of spacetime. Somewhat similar constructions have been developed for pilot wave theories in order to bring them more in line with the spirit of relativity. See [38].

<sup>11</sup>Situations in which several sets of interacting fields are superposed in the same spacetime region (somewhat like Hardy's paradox) can be handled in a similar fashion.

<sup>12</sup>In simple, idealized cases this decomposition remains the same throughout the measurement, but, in general, it can change due to changes in the interactions that occur or due to the random evolution of  $\sigma(s)$ .

defined.<sup>13</sup>

To fully develop this approach from a theoretical standpoint one would start by viewing projection as a continuous process *in Hilbert space* (with respect to the sequencing parameter,  $s$ ). The goal would be to recover a Lorentz invariant formulation by developing a suitable method of averaging over the possible families of evolving spacelike hypersurfaces and over the possible sequences that are consistent with various outcomes. However, the goal here is not to develop a full Lorentz invariant theory. It is, rather to derive enough properties that such a theory must possess in order to assess the possibility of *experimental* tests of wave function collapse. For this purpose, let us assume that some effective averaging procedure can be found that would allow us to view elementary interactions as distinct individual events, possibly accompanied by small, discrete increments,  $\pm\delta$ , in the squared amplitudes,  $\alpha\alpha^*$  and  $\beta\beta^*$ . Based on this assumption, the next section will outline an experimental strategy.

## 6 An Experimental Approach

The hypothesis that there is some amplitude shift associated with every nonlocal, entangling interaction suggests the possibility that some (small) deviations from perfectly linear evolution might be observable in entangled systems consisting of just a few elementary particles. Although the hypothesis is strictly phenomenological, it is derived from very general considerations based on the no-signaling principle. This principle can be interpreted in *purely physical terms* as the prohibition of superluminal information transmission, *if* it assumed that projection is a real, and, hence, non-deterministic effect arising from elementary processes. The possibility that this simple chain of inferences from fundamental principles might provide a coherent account of the macro-micro connection makes its experimental consequences worth exploring.

One might be skeptical about the feasibility of testing for deviations from strictly linear evolution since some of the demonstrations of the persistence of superposition have dealt with entangled systems that approach macroscopic, or at least, mesoscopic scale[52, 55, 56], as measured by the number of elementary particles involved. But the hypothesis presented here suggests that a more relevant measure of macroscopicity is the number of entangling interactions experienced by any elementary particles in generating the entangled state. Experiments have demonstrated superposition in a wide variety of contexts [22, 23, 24, 25, 26, 52, 55, 56, 57, 58], but, in every case the chains of interactions that generate the entanglement relations are fairly short. Even when large numbers of particles are involved, none of them experience more

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<sup>13</sup>Because the definition of reference frames depends on the same nondeterministic processes that generate the nonlocal quantum effects, no finite speed can be attributed to the nonlocal effects in any reference frame. This is how the hypothesis of an incremental collapse process can be reconciled with the recent demonstration by Bancal, et al., [53, 54] that no influences propagating at any finite speed (even a superluminal speed) can explain nonlocal quantum correlations.



than a few interactions.<sup>14</sup> If this were not the case the decoherence induced by the interactions would make it unfeasible to observe these entangled states.

To devise an experimental strategy, let us recall that the proposed hypothesis suggests that the projection process has the character of a random walk with end points. If the average shift in squared amplitude is  $\delta_{ave}$ , then, in a situation with two approximately equal branches ( $\alpha\alpha^* \approx \beta\beta^*$ ), it would require roughly  $1/(4\delta_{ave}^2)$  elementary interactions to bring about complete collapse, and the possible deviation from linearity in a single interaction would be of the order,  $\delta_{ave}^2$ . So, for example, if  $\delta_{ave} = 0.01$ , after 40 interactions the average deviation from linearity would only be about  $\sqrt{40}/(0.01^2) \approx 0.0064$ . Clearly, such small effects would be very hard to see.<sup>15</sup> The most obvious way to get around this problem is to start the random walk close to one of the end points. If  $\alpha\alpha^* \approx \delta_{ave}$  then the size of the possible deviations increases from  $\delta_{ave}^2$  to about the order of  $\delta_{ave}$ .

The strategy must also balance two competing considerations. On the one hand, given the presumed smallness of the effect, one would like to use elementary systems that can be very precisely controlled. On the other hand, since projection is assumed to arise from entangling interactions, it is desirable to use particles such as electrons that interact very readily, and survive the interaction. The recent progress in designing experiments that involve the use of an electronic Mach-Zehnder interferometer (EMZI)[59, 60, 61, 62, 63, 64] provides a reason to believe that it will eventually be feasible to use electrons, and still maintain sufficient coherence of the states to make the necessary measurements. So the approach outlined here will be based on a variation of the coupled EMZI version of a quantum eraser.

In order to have a reasonable chance of observing any deviations from linearity, it is important to develop a scheme to enhance the very small signal. For this purpose, consider a completely idealized coupled double-slit quantum eraser. Suppose that an electron wave function is split into two branches of equal magnitude, and that one of the branches interacts with a second electron wave function. Assume that the experimental apparatus is designed so that the second, "detector" electron is split into two well-defined, orthogonal branches by the interaction with one branch of the first electron. This can be represented as follows.

$$\begin{aligned} & (1/\sqrt{2})(|I_1\rangle + |N_1\rangle) \otimes |X_2\rangle \\ \Rightarrow & (1/\sqrt{2})(|I_1\rangle|I_2\rangle + |N_1\rangle|N_2\rangle) = (1/\sqrt{2})(|S_1\rangle|S_2\rangle + |A_1\rangle|A_2\rangle). \end{aligned} \quad (6.1)$$

In this expression, 1 and 2 refer to the first and second electron,  $|I\rangle$  and  $|N\rangle$  refer to interacting and noninteracting components of the wave function, and  $|S\rangle$  and  $|A\rangle$  refer

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<sup>14</sup>Even the exotic states of matter that are studied in solid state physics[55] that consist of  $10^6$  to  $10^{12}$  particles are built up in a *pairwise* fashion with the components doubling in size at each stage. Since  $10^{12} \approx 2^{40}$ , no portion of the final entangled system undergoes more than a few dozen interactions.

<sup>15</sup>This also explains why the experiments done to date, which are looking for the persistence of superposition, have not seen any evidence of deviations from linearity.

to symmetric and antisymmetric superpositions of the interacting and noninteracting components:  $|S\rangle = (1/\sqrt{2})(|I\rangle + |N\rangle)$  and  $|A\rangle = (1/\sqrt{2})(|I\rangle - |N\rangle)$ . Subsequent to the interaction that creates the entangled state, the branches of each electron wave function are steered into double-slit arrangements in a synchronized manner. The position of the electrons at each receptor screen is recorded and time-stamped as they are detected. After a sufficient sample of correlated pairs is registered, the statistics can be analyzed. The overall distributions can be represented as the sum of probabilities associated with either the  $|I_1\rangle|I_2\rangle$  and  $|N_1\rangle|N_2\rangle$  states *or* the sum associated with the  $|S_1\rangle|S_2\rangle$  and  $|A_1\rangle|A_2\rangle$  states.<sup>16</sup>

Since the "measurement" interaction was made in the  $|I\rangle$   $|N\rangle$  basis, one expects that a decomposition of the total distributions into  $|I\rangle$  and  $|N\rangle$  subsets would demonstrate the correlations between the electrons that were established by the interaction. In other words, the electrons in the first apparatus that are consistent with the probability distribution associated with the  $|I\rangle$  branch of the wave function would be correlated with electrons in the second apparatus that are also consistent with the  $|I\rangle$  branch distribution, and the same would hold for the  $|N\rangle$  branches. This would be true whether or not the interaction had brought about a projection to one of the branches. If there were an actual projection to one of the branches, then the entanglement between the electrons would be broken, and correlations would not be visible in a decomposition corresponding to the complementary basis.

Since we do not expect that a single elementary interaction is capable of completely collapsing the wave function in this situation, we should also be able to observe the correlations between the electrons on the two receptor screens if we decompose the the data sets into  $|S\rangle$  and  $|A\rangle$  distributions. The EMZI experiment mentioned above has confirmed this expectation[64].

The situation just described assumes that the two branches of the wave function are equal in magnitude. One could now alter the experimental design so that the squared amplitude of the interacting branch was approximately equal to one's best estimate of  $\delta_{ave}$ . The interaction changes the state in the following manner:

$$(\alpha|I_1\rangle + \beta|N_1\rangle) \otimes |X_2\rangle \implies \alpha'|I_1\rangle|I_2\rangle + \beta'|N_1\rangle|N_2\rangle. \quad (6.2)$$

The change in amplitudes from  $\alpha$  and  $\beta$  to  $\alpha'$  and  $\beta'$  reflects the possible shift in amplitudes associated with the interaction.

If the incremental collapse hypothesis is correct, one would expect that there would be a projection to the larger branch for roughly half of the individual measurements. This would imply a change in some of the correlation statistics, but this change would be extremely difficult to see. Given the assumed small values of  $\delta_{ave}$ , the overall pattern would be dominated by the large branch, and would look very much like a single-slit probability distribution. The very small signal would be buried in experimental noise.

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<sup>16</sup>The probability distributions corresponding to the  $|S\rangle$  and  $|A\rangle$  states are double-slit interference patterns that are completely out of phase.

To get around this problem, after the initial interaction one could split off a second small interacting branch from the large noninteracting branch of first electron, with amplitude,  $\gamma$ , with a magnitude equal to that of  $\alpha$ :  $\gamma\gamma^* \approx \alpha\alpha^* \approx \delta_{ave}$ . This second small branch would then be induced to interact with the  $|N\rangle$  component of the "measurement" electron. The overall development of the system would look like:

$$\begin{aligned} (\alpha|I_{1a}\rangle + \beta_a|N_{1a}\rangle) \otimes |X_2\rangle &\Longrightarrow \alpha'|I_{1a}\rangle|I_{2a}\rangle + \beta'_a|N_{1a}\rangle|N_{2a}\rangle \Longrightarrow \\ \alpha'|I_{1a}\rangle|I_{2a}\rangle + (\gamma|I_{1b}\rangle + \beta_b|N_{1b}\rangle) \otimes |N_{2a}\rangle &\Longrightarrow \\ \alpha'|I_{1a}\rangle|I_{2a}\rangle + \gamma'|I_{1b}\rangle|I_{2b}\rangle + \beta'_b|N_{1b}\rangle|N_{2b}\rangle. \end{aligned} \quad (6.3)$$

Subscripts 'a' and 'b' have been attached to distinguish the first interacting branch from the second. As before, the primes on the amplitudes,  $\alpha$ ,  $\beta$ , and  $\gamma$  indicate possible shifts brought about by the interactions; the change in subscripts from  $\beta_a$  to  $\beta_b$  represents the splitting of the noninteracting branch into a small ( $\gamma$ ) component and a large ( $\beta_b$ ) component.<sup>17</sup>

The strategy would now be to steer the two different interacting branches, 'a' and 'b', of the first electron into a double slit arrangement, and the interacting branches of the second ("measurement") electron into a similar apparatus. One would then examine the correlations in the resulting distributions. The  $|N_{1b}\rangle|N_{2b}\rangle$  branches (which should be perfectly correlated) could be steered away from the double slit screens and measured separately. Since, by design, the magnitude of the noninteracting branches is much greater than that of the interacting branches, this would mean that one is filtering out the large majority of the experimental data so that it does not cloud the essential signal. Clearly the most challenging aspects of the experiment would be the design, construction, and calibration of the circuits and measurement system; the additional burden resulting from the need for a very large number of experimental runs would probably not greatly increase the overall difficulty of carrying out this sort of test.

The process outlined in 6.3 would yield the three-branch entangled state:  $\alpha'|I_{1a}\rangle|I_{2a}\rangle + \gamma'|I_{1b}\rangle|I_{2b}\rangle + \beta'_b|N_{1b}\rangle|N_{2b}\rangle$ , but we are interested only in the first two branches:  $\alpha'|I_{1a}\rangle|I_{2a}\rangle + \gamma'|I_{1b}\rangle|I_{2b}\rangle$ . Since our interest is now restricted to these states, let us define symmetric and antisymmetric combinations of just those components:  $|S'\rangle = (1/\sqrt{2})(|I_{1a}\rangle + |I_{1b}\rangle)$  and  $|A'\rangle = (1/\sqrt{2})(|I_{1a}\rangle - |I_{1b}\rangle)$ . With these definitions we can rewrite the preceding expression as:

$$\begin{aligned} \alpha'|I_{1a}\rangle|I_{2a}\rangle + \gamma'|I_{1b}\rangle|I_{2b}\rangle &= \\ (1/2)[\alpha'(|S'_1\rangle + |A'_1\rangle)(|S'_2\rangle + |A'_2\rangle) + \gamma'(|S'_1\rangle - |A'_1\rangle)(|S'_2\rangle - |A'_2\rangle)] &= \\ (1/2)[(\alpha' + \gamma')(|S'_1\rangle|S'_2\rangle + |A'_1\rangle|A'_2\rangle) + (\alpha' - \gamma')(|S'_1\rangle|A'_2\rangle + |A'_1\rangle|S'_2\rangle)]. \end{aligned} \quad (6.4)$$

To evaluate this expression we can first assume that the relative phase differences between  $\alpha$  and  $\gamma$  can be controlled in the experiment, so that only differences in

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<sup>17</sup>The second interaction involving  $\gamma|I_{1b}\rangle$  would change  $\alpha'$ , but only by a negligible amount since both  $\alpha'|I_{1b}\rangle$  and  $\gamma|I_{1b}\rangle$  constitute a very small fraction of the total wave function.

magnitude matter.<sup>18</sup> Then if  $\gamma = \alpha$  and there are no amplitude shifts induced by the interactions (i.e., no breakdown of superposition), the cross terms,  $|S_1'\rangle|A_2'\rangle$  and  $|A_1'\rangle|S_2'\rangle$  would be zero. Nonzero cross terms would be the signature of wave function collapse (again, assuming that *strict* equality holds between the initial values of  $\alpha$  and  $\gamma$ ).

To estimate the size of a possible collapse signal, let us begin with the ideal situation:  $\gamma\gamma^* = \alpha\alpha^* = \delta_{ave}$ . Let us also assume that the size of the possible shifts,  $\delta$ , remains the same for interactions that are essentially identical.

Given the incremental collapse hypothesis, in approximately one half of the initial interactions the squared amplitude of the  $|I_{1a}\rangle|I_{2a}\rangle$  component would be changed to zero, and in the other half it would be doubled. The same would be true for the  $|I_{1b}\rangle|I_{2b}\rangle$  component in the second interaction. So, the four possibilities are: (a) the 'a' component would be enhanced and the 'b' component would be zeroed out; (b) the 'b' component would be enhanced and the 'a' component would be zeroed out; (c) both interacting components would be enhanced; and (d) both interacting components would be zeroed out. Each of these cases should occur 1/4 of the time. Since each of the two branches initially has a squared amplitude of  $\delta$ , the overall probability of the entangled electrons being registered in the coupled double slit detectors should be  $2\delta$ . The electrons should be found in the noninteracting branches the rest of the time.

Although cases 'a', 'b', and 'c' occur with the same frequency, their probabilities of subsequent detection in the coupled double slit apparatus are not the same. Since  $\alpha'\alpha'^* = \alpha\alpha^* \pm \delta$ ,  $\gamma'\gamma'^* = \gamma\gamma^* \pm \delta$ , and  $\gamma\gamma^* = \alpha\alpha^* = \delta_{ave}$ , we get  $\alpha'\alpha'^* = 2\delta$  or 0 and the same holds for  $\gamma'\gamma'^*$ . Since it is also assumed that the relative phases are properly controlled, we can redefine both  $\alpha'$  and  $\gamma'$  in terms of their magnitudes:  $\alpha', \gamma' = \sqrt{2\delta}$ , or  $\alpha', \gamma' = 0$ . Plugging these values into the expression from 6.4 for the three cases, 'a', 'b', and 'c' yields:

- (a)  $(1/2)[(\sqrt{2\delta} + 0)(|S_1'\rangle|S_2'\rangle + |A_1'\rangle|A_2'\rangle) + (\sqrt{2\delta} - 0)(|S_1'\rangle|A_2'\rangle + |A_1'\rangle|S_2'\rangle)];$
- (b)  $(1/2)[(0 + \sqrt{2\delta})(|S_1'\rangle|S_2'\rangle + |A_1'\rangle|A_2'\rangle) + (0 - \sqrt{2\delta})(|S_1'\rangle|A_2'\rangle + |A_1'\rangle|S_2'\rangle)];$
- (c)  $(1/2)[(\sqrt{2\delta} + \sqrt{2\delta})(|S_1'\rangle|S_2'\rangle + |A_1'\rangle|A_2'\rangle) + (\sqrt{2\delta} - \sqrt{2\delta})(|S_1'\rangle|A_2'\rangle + |A_1'\rangle|S_2'\rangle)].$

These simplify to:

- (a)  $(1/\sqrt{2})\sqrt{\delta}[(|S_1'\rangle|S_2'\rangle + |A_1'\rangle|A_2'\rangle) + (|S_1'\rangle|A_2'\rangle + |A_1'\rangle|S_2'\rangle)];$
- (b)  $(1/\sqrt{2})\sqrt{\delta}[(|S_1'\rangle|S_2'\rangle + |A_1'\rangle|A_2'\rangle) - (|S_1'\rangle|A_2'\rangle + |A_1'\rangle|S_2'\rangle)];$
- (c)  $\sqrt{2\delta}[(|S_1'\rangle|S_2'\rangle + |A_1'\rangle|A_2'\rangle)].$

Noting that the four states,  $|S_1'\rangle|S_2'\rangle, |A_1'\rangle|A_2'\rangle, |S_1'\rangle|A_2'\rangle, |A_1'\rangle|S_2'\rangle$ , are mutually orthogonal, the probabilities can be readily calculated. The squared expressions for cases 'a' and 'b' each give a value of  $\delta/2$  for each of the four possible states. Squaring the expression for case 'c' yields  $2\delta$  for each of the two states that are consistent with continued superposition:  $|S_1'\rangle|S_2'\rangle$  and  $|A_1'\rangle|A_2'\rangle$ . Since each of these cases occurs 1/4 of the time, the total combined probability for  $|S_1'\rangle|S_2'\rangle$  and  $|A_1'\rangle|A_2'\rangle$  is  $(3/2)\delta$ , and the

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<sup>18</sup>The preservation of relative phases in any amplitude shifts from  $\alpha$  and  $\gamma$  to  $\alpha'$  and  $\gamma'$  follows from the need to prevent superluminal signaling.

probability for the cross terms,  $|S'_1\rangle|A'_2\rangle$  and  $|A'_1\rangle|S'_2\rangle$ , is  $(1/2)\delta$ . So, overall, roughly one quarter of the electron pairs that are not detected in the noninteracting branches should fall in different states with respect to the  $|S'\rangle|A'\rangle$  decomposition. This would be the signature of genuine wave function collapse for this arrangement.

Since, at this point, we can only make very crude guesses about the actual value of  $\delta_{ave}$  it is important to evaluate the possible signal of wave function collapse for situations in which  $\alpha\alpha^*$  and  $\gamma\gamma^*$  differ substantially from  $\delta_{ave}$ . Note first, that since  $\delta$  cannot exceed either  $\alpha\alpha^*$  or  $\gamma\gamma^*$ , if  $\alpha\alpha^*$  and  $\gamma\gamma^*$  are less than  $\delta_{ave}$ , the analysis would proceed exactly as above, and the relative signature of deviations from linearity would be the same as described in the previous paragraph. However, more experimental runs would be required to accumulate the same amount of data.

To determine the observability of deviations in situations in which  $\alpha\alpha^*$  and  $\gamma\gamma^*$  are greater than  $\delta_{ave}$  we need to return to the general expression from 6.4:

$$(1/2)[(\alpha' + \gamma')(|S'_1\rangle|S'_2\rangle + |A'_1\rangle|A'_2\rangle) + (\alpha' - \gamma')(|S'_1\rangle|A'_2\rangle + |A'_1\rangle|S'_2\rangle)].$$

Again we assume that the initial values of  $\alpha\alpha^*$  and  $\gamma\gamma^*$  are equal by experimental design, and we ignore relative phases. We can define  $r \equiv \alpha\alpha^*/\delta_{ave}$ , so that  $\alpha = \gamma = \sqrt{r\delta_{ave}}$ . Since  $\alpha' = \sqrt{\alpha \pm \delta_{ave}}$  and  $\gamma' = \sqrt{\gamma \pm \delta_{ave}}$ , substitution into the expression above yields the following amplitude:

$$(1/2)[(\sqrt{r\delta_{ave} \pm \delta_{ave}} + \sqrt{r\delta_{ave} \pm \delta_{ave}})(|S'_1\rangle|S'_2\rangle + |A'_1\rangle|A'_2\rangle) + (\sqrt{r\delta_{ave} \pm \delta_{ave}} - \sqrt{r\delta_{ave} \pm \delta_{ave}})(|S'_1\rangle|A'_2\rangle + |A'_1\rangle|S'_2\rangle)]. \quad (6.5)$$

As before there are four possible cases since  $\alpha'$  and  $\gamma'$  can either increase or decrease, independently, from their initial values. The total probability of the four cases is:  $\alpha\alpha^* + \gamma\gamma^* = 2r\delta_{ave}$ . Cases 'c' and 'd', in which both branches are either enhanced or decreased in amplitude, give a zero probability for the cross terms,  $|S'_1\rangle|A'_2\rangle$  and  $|A'_1\rangle|S'_2\rangle$ . Cases 'a' and 'b' each contribute a probability of  $(1/4)(\delta_{ave}/4)(\sqrt{r+1} - \sqrt{r-1})^2 = (1/8)\delta_{ave}(r - \sqrt{r^2 - 1})$  for *each* of the cross terms, giving a combined probability of  $(1/2)\delta_{ave}(r - \sqrt{r^2 - 1})$  for the occurrence of the cross terms. (The leading factor of 1/4 comes from the frequency of each case.) The ratio of the cross term probability to the total probability provides the measure of deviations from perfect linearity. Its value is  $(r - \sqrt{r^2 - 1})/4r$ .

One can readily verify that for  $r = 1$ , the expression reproduces the value for the relative signal of 1/4 that was derived above, but for values of  $r$  greater than 1 the signal decreases rapidly. For  $r = 2$ , it drops below 0.07, and with  $r = 10$ , it is just slightly greater than 0.01. So the possible deviations from linearity could be difficult to see, and, even with precisely controlled experiments, they will not become visible unless the squared amplitudes of the interacting branches are close to the average shift value,  $\delta_{ave}$ . This point highlights some of the challenges involved in carrying out an experiment of this kind; but the sharp increases in the observability of wave function collapse as one approaches  $\delta_{ave}$  might also provide an additional signature of deviations from linear evolution. In fact, this sharp increase in violations of quantum coherence near  $\delta_{ave}$  could provide evidence of wave function collapse, even if the

primary signature is reduced by experimental noise.

The discussion up to this point has involved a completely idealized arrangement. A possible implementation could be a variation of the coupled EMZI experiment carried out by Weisz, et al.[64]. They describe the the standard EMZI set-up as follows:

”An electronic MZI is formed by manipulating quasi one dimensional, chiral edge channels, which are formed in the integer quantum Hall effect regime. Such a realization allows directing the path of electrons at will - leading to high visibility interference pattern. Potential barriers, formed by quantum point contacts (QPCs), take the role of optical beam splitters, transmitting and reflecting impinging electrons with amplitudes  $t_i$  and  $r_i$ , respectively, with  $|t_i|^2 + |r_i|^2 = 1$  .”

The wave functions of the electrons are split into two branches which traverse alternate paths around a loop enclosing a strong magnetic field. The relative phases of the branches are controlled through the Aharonov-Bohm effect. For more details see the previously cited sources[59, 60, 62, 63, 64].

The experiment employed a symmetric arrangement of coupled EMZI's, each with one interacting and one noninteracting branch. All of the branches were of equal magnitude. Because of the complete symmetry, the distinction between EMZI's labeled as the ”system interferometer” and ”path detector” is completely arbitrary.

To more closely match this configuration, let us modify the arrangement described in the idealized description to drop the distinction between the ”measured” electron (1), and the ”measurement” electron (2). Rather than splitting off two small branches from the same electron, one could split off a small branch from each electron, and induce it to interact with the large branch of the other electron. The process described by expression 6.3, which resulted in the state:

$\alpha'|I_{1a}\rangle|I_{2a}\rangle + \gamma'|I_{1b}\rangle|I_{2b}\rangle + \beta'_b|N_{1b}\rangle|N_{2b}\rangle$ , would be replaced by

$$\begin{aligned} & ( \alpha_a|J_{1a}\rangle + \beta_b|M_{1b}\rangle ) \otimes ( \alpha_b|J_{2b}\rangle + \beta_a|M_{2a}\rangle ) \implies \\ & (\alpha_a\beta_a)'|J_{1a}\rangle|K_{2a}\rangle + \beta'_b|M_{1b}\rangle \otimes ( \alpha'_b|J_{2b}\rangle + \beta'_a|M_{2a}\rangle ) \implies \\ & (\alpha_a\beta_a)'|J_{1a}\rangle|K_{2a}\rangle + (\alpha_b\beta_b)'|K_{1b}\rangle|J_{2b}\rangle + (\beta_b\beta_a)'|M_{1b}\rangle|M_{2a}\rangle + (\alpha_a\alpha_b)'|J_{1a}\rangle|J_{2b}\rangle \end{aligned} \quad (6.6)$$

The numerical subscripts, 1 and 2 label the two electrons (as before); the lower case subscripts, 'a' and 'b', label the different interactions (as before). The labeling of the branches has changed from  $|I\rangle$  and  $|N\rangle$  to  $|J\rangle$ ,  $|M\rangle$ , and  $|K\rangle$  to facilitate the discussion of some issues raised by the new arrangement. 'J' labels the smaller branches from the initial splitting of the wave function, and 'M' labels the larger ones. 'K' labels the (small) branches that are separated from the large ones by the interaction. The first three terms in the new expression correspond to the three terms in 6.3. Since  $|\beta|$  is very close to 1, one can see that the magnitudes do not differ by very much. The last term,  $(\alpha_a\alpha_b)'|J_{1a}\rangle|J_{2b}\rangle$  is new. It is very small since the coefficient is the product of two small terms, and so it will not significantly affect the statistics.

The interactions are shown in two stages because it is desirable for them to be timelike separated. The reason for this is that the derivation of the collapse signature above assumed that the possible changes in the amplitudes associated with the interactions were independent. This requires that they occur at different values of the sequencing parameter,  $s$ , described in section 5. If the interactions are simultaneous (in some reference frame) and in close proximity, there is an increased probability that this assumption will be violated.

Although this more symmetric form corresponds more closely to the actual coupled EMZI experiment, there are important differences. The actual experiment involved branches that were equal in magnitude, with a single interaction. The most important difference is, probably, that the large 'M' branch and the smaller 'K' branch (separated from 'M' by the interaction) must be steered to different detectors.

Of these three differences the simplest to address is the relative size of the coefficients. The QPC's, which function as beam splitters, can be readily adjusted to vary the reflection and transmission coefficients. This has already been done. Some complications might arise if  $\delta_{ave}$  is extremely small, but this can only be determined by experiment.

The need for a second, timelike-separated, interaction poses a more serious challenge. This would involve a somewhat longer and more complicated circuit. This might pose substantial technical challenges.

The biggest obstacle is the need to steer the 'M' and 'K' branches along diverging paths. In the present EMZI design, there is a displacement due to the interaction, and a resulting separation in phase from the Aharonov-Bohm effect. But there is not a clean spatial separation that allows the two different branches to be guided to different detectors. To have a reasonable chance of observing the possible collapse signature described above, some means of clearly distinguishing 'M' and 'K' branches must be developed.

Clearly, the tests proposed here would be challenging for a number of reasons. The design and building of the apparatus, and the execution of the experiment would require extraordinary skill and effort. But the success of Weisz, et al.[64] in implementing an electronic quantum eraser provides reasonable hope that such an experiment is feasible.

## 7 Discussion

Bell showed that some experimental predictions of quantum theory are dependent on where one places the boundary between the measured system and the measuring apparatus. This ambiguity is the central problem in understanding quantum theory. One can "shut up and calculate" [65] only as long as the results of those calculations do not contradict one another.

To resolve the inconsistency pointed out by Bell, we must, at some point, de-

termine whether projection is a real physical phenomenon. The demonstrations of superposition in systems that have become entangled by interactions span an impressive range of physical situations[22, 23, 24, 25, 26, 52, 55, 56, 57, 58], and they are important first steps in determining the extent of linear evolution. However, to establish the complete absence of any real projection effects would require something like the building of a perfectly functioning quantum computer of arbitrarily large complexity, or the teleportation of extremely complicated quantum states. Clearly, these tasks are far beyond any foreseeable technology.

To complement the efforts to exhibit superposition in ever more complex circumstances, one can approach this issue by taking seriously the possibility that the wave function (understood as a genuine physical entity) really does collapse, try to discern how this could be connected to fundamental processes, and determine what experimental predictions follow from the hypothesized connection. That is the approach that has been advocated here.

To pursue this approach, what is needed is a change in our view about the relationship of nonlocal effects to relativity. Ultimately, quantum mechanics and relativity face the test of experiment as a *single* body of principles. So the challenge is not to embed a description of projection into a Lorentz invariant spacetime. It is, rather, to incorporate nonlocality into a Lorentz invariant *theory*.

The crucial feature of contemporary theory that allows us to describe quantum systems in a relativistic framework is local commutativity, the formal expression of the no-superluminal-signaling principle. This is the logical starting point for analyses that aim to explain nonlocal phenomena like projection in terms of fundamental processes. No-signaling implies that any deterministic description of quantum evolution must be linear, and that any fundamental account of projection must be stochastic[19]. This essential nondeterministic aspect of elementary processes implies that there is some lower limit on the size and complexity of systems that can instantiate reproducible, transmittable information. The lack of complete determinism can thus explain both the no-signaling principle and how the additional spacetime structure needed to accommodate the nonlocal, nondeterministic effects can remain, in principle, unobservable.

With the limits on physical instantiations of information, it is sensible to define the no-signaling principle as the prohibition of superluminal information transmission. This definition uses only basic physical terms, and avoids any essential reference to intelligent observers. Although the concepts of information and signaling cannot be applied directly to elementary particles, various aspects of the behavior of those particles can be captured as information by more complex physical systems. These aspects include the nonlocal, probabilistic changes of state that occur in measurement-like interactions. This is why the no-signaling principle can impose specific constraints on how those changes can occur. Among the most notable of those constraints is the implication that projection must originate in the nonlocally entangling interactions that constitute the problematic measurement processes.



The most straightforward application of this constraint suggests that the elementary interactions that generate nonlocal entanglement relations include a small nondeterministic transfer of amplitude between the interacting and noninteracting components of the wave function. Although these small deviations from perfectly linear evolution usually tend to be hidden by decoherence effects, there are experimental arrangements that might reveal them. One such arrangement has been outlined here.

## References

- [1] Schlosshauer, M., Kofler, J., Zeilinger, A.: A Snapshot of Foundational Attitudes Toward Quantum Mechanics. arXiv:1301.1069v1 [quant-ph] (2013)
- [2] Norsen, N., Nelson, S.: Yet Another Snapshot of Foundational Attitudes. arXiv:1306.4646v2 [quant-ph] (2013)
- [3] Bell, J.S.: On wave packet reduction in the Coleman-Hepp model. *Helv. Phys. Act.* **48**, 93 (1975). Reprinted in **Speakable and Unspeakable in Quantum Mechanics**, Revised edition (2004) pp. 48/49
- [4] Hepp, K.: *Helv. Phys. Act.* **45**, 237 (1972)
- [5] Bohr, N.: Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **48**, 696 (1935)
- [6] Heisenberg, W.: *Wandlungen in den Grundlagen der Naturwissenschaft*. S. Hirzel Verlag, Zurich (1949)
- [7] Bacciagaluppi, G., Crull, E.: Heisenberg (and Schrödinger, and Pauli) on Hidden Variables. *Stud. Hist. Phil. Mod. Phys.* **40**, 374 (2009)
- [8] Everett, H.: Relative state formulation of quantum mechanics. *Rev. Mod. Phys.* **29**, 454 (1957)
- [9] Hartle, J.B.: The quasiclassical realms of this quantum universe. arXiv:0806.3776v3 [quant-ph] (2008)
- [10] de Broglie, L.: *Tentative d'interpretation causale non-lineaire de la mecanique ondulatoire*. Gautier-Villars, Paris, (1956)
- [11] Bohm, D.: A suggested interpretation of quantum theory in terms of 'hidden' variables, I. *Phys. Rev.* **85**, 166-179 (1952)
- [12] Bohm, D., Hiley, B. J.: *The Undivided Universe: An Ontological Interpretation of Quantum Theory*. Routledge, New York (1993)

- [13] Ghirardi, G.C., Rimini, A., Weber, T.: Unified dynamics for microscopic and macroscopic systems. *Phys. Rev.* **D34**, 470-491 (1986)
- [14] Bassi, A., Ghirardi, G.C.: Dynamical Reduction Models. *Phys. Rep.* **379**, 257-427 (2003)
- [15] Pearle, P.: How stands collapse I. arXiv:quant-ph/0611211v1 (2006)
- [16] Pearle, P.: How stands collapse II. arXiv:quant-ph/0611212v3 (2007)
- [17] Bedingham, D.J.: Dynamical state reduction in an EPR experiment. arXiv:0907.2327v1 [quant-ph] (2009)
- [18] Bell, J.S.: Speakable and Unspeakable in Quantum Mechanics. Introductory remarks at Naples-Amalfi meeting, May 7, 1984. Reprinted in **Speakable and Unspeakable in Quantum Mechanics**, Revised edition (2004) p. 171
- [19] Gisin, N.: Stochastic quantum dynamics and relativity. *Helv. Phys. Act.* **62**, 363 (1989)
- [20] Gisin, N.: Weinberg's Non-Linear Quantum Mechanics And Supraluminal Communications. *Phys. Lett. A* **143**, 1 (1990)
- [21] Maudlin, T.: Quantum Non-Locality and Relativity. 2nd Edition, Blackwell Publishers Inc., Malden (2002)
- [22] Scully, M.O., Druhl, K.: Quantum eraser: A proposed photon correlation experiment concerning observation and "delayed choice" in quantum mechanics. *Phys. Rev.* **A25**, 2208 (1982)
- [23] Scully, M.O., Englert, B.G., Schwinger, J.: Spin coherence and Humpty-Dumpty. III. The effects of observation. *Phys.Rev.* **A40**, 1775 (1989)
- [24] Scully, M.O., Englert, B.G., Walther, H.: Quantum Optical Tests of Complementarity. *Nature* **351**, 111, London (1991)
- [25] Scully, M.O., Kim, Y-H, Kulik, S.P., Shih, Y.H.: A Delayed Choice Quantum Eraser. *Phys. Rev. Lett.* **84**, 1-5 (2000)
- [26] Walborn, S., Terra Cunha, M.O., Padua, S., Monken, C.H.: Double-slit quantum eraser. *Phys. Rev.* **A65**, 033818 (2002)
- [27] Mermin, N.D.: **Quantum Computer Science: An Introduction**, Cambridge University Press, (2007)
- [28] Gillis, E.J.: Causality, Measurement, and Elementary Interactions. *Foundations of Physics* **41**, 1757 (2011)

- [29] Einstein, A.: On the Electrodynamics of Moving Bodies. *Annalen der Physik* **17**, 891 (1905)
- [30] Born, M.: On the quantum mechanics of collisions. *Zeitschrift fur Physik* **37**, 863-67 (1926)
- [31] Wootters, W.K., Zurek, W.H.: A single quantum cannot be cloned. *Nature* **299**, 802-803 (1982)
- [32] Einstein, A.: Geometry and Experience(address to the Prussian Academy of Sciences on January 27,1921). Reprinted in **Ideas and Opinions**, Dell Publishing Co.,Inc. New York (1981)
- [33] Bell, J.S.: On the Einstein Podolsky Rosen paradox. *Physics* **1**, 195 (1964)
- [34] Bell, J.S.: On the problem of hidden variables in quantum mechanics. *Rev. Mod. Phys.* **38**, 447 (1966)
- [35] Aspect, A., Grangier, P., Roger, G.: Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities. *Phys.Rev.Lett* **49**, 91 (1982)
- [36] Aspect, A., Dalibard, J., Roger, G.: Experimental Test of Bell's Inequalities Using Time-Varying Analyzers. *PhysRevLett.* **49**, 1804 (1982)
- [37] Bell, J.S.: *Speakable and Unspeakable in Quantum Mechanics*. 2nd Edition, Cambridge University Press, New York (2004)
- [38] Durr, D., Goldstein, S., Norsen, T., Struyve, W., Zanghi, N.: Can Bohmian mechanics be made relativistic? arXiv:1307.1714v1 [quant-ph] (2013).
- [39] Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of reality be considered complete?.
- [40] Bell, J.S.: La Nouvelle Cuisine. In: A. Sarlemijn and P. Kroes, (eds.) *Between Science and Technology*, p.000. Elsevier Science Publishers, Oxford (1990)
- [41] Norsen, T.: J. S. Bell's Concept of Local Causality. arXiv:0707.0401v2 [quant-ph] (2010).
- [42] Gleason, A.M.: Measures on the closed subspaces of a Hilbert space. *Journal of Mathematics and Mechanics* **6**, 885-893 (1957)
- [43] Elitzur, A. C.: Locality and indeterminism preserve the second law. *Phys. Lett. A* **167**, 335 (1992)

- [44] Popescu, S., Rohrlich, D.: Quantum Nonlocality as an Axiom. *Foundations of Physics* **24**, 379 (1994)
- [45] Elitzur, A.C., Dolev, S.: Quantum Phenomena within a New Theory of Time. In Elitzur, A.C., et al. (eds.) *Quo Vadis Quantum Mechanics?* Springer, Berlin Heidelberg (2005)
- [46] Elitzur, A.C., Dolev, S.: Becoming as a Bridge Between Quantum Mechanics and Relativity. In R.Buccheri et al. (eds.) *Endophysics, Time, Quantum, and the Subjective*, pp. 197-214. World Scientific Publishing Co., Singapore (2005)
- [47] Zhang, Q.-R.: Statistical Separability And The Consistency Between Quantum Theory, Relativity, And The Causality. arXiv:quant-phys/0512150v1 (2005)
- [48] Masanes, Ll., Acin, A., Gisin, N.: General Properties of Nonsignaling Theories. *Phys. Rev.* **A73**, 012112 (2006)
- [49] Svetlichny, G.: Long Range Correlations and Relativity: Metatheoretic Considerations. arXiv:quant-ph/9902064v1 (1999)
- [50] Svetlichny, G.: Causality implies formal state collapse. *Foundations of Physics* **33**, 641 (2003)
- [51] Bohr, N.: : *Atomic Physics and Human Knowledge*. John Wiley & Sons, New York (1958)
- [52] O'Reilly, E.J., Olaya-Castro, A.: Non-classicality of the molecular vibrations assisting exciton energy transfer at room temperature. *Nature Communications* **5**, 3012, (2014)
- [53] Bancal, J-D., Pironio, S., Acin, A., Liang, Y-C., Scarani, V., Gisin, N.: Quantum non-locality based on finite-speed causal influences leads to superluminal signalling. *Nature Physics* **8**, 867 (2012)
- [54] Gisin, N.: Quantum correlations in Newtonian space and time: arbitrarily fast communication or nonlocality. arXiv:1210.7308v2 [quant-ph] (2012)
- [55] Sachdev, S.: The quantum phases of matter. arXiv:hep-th 1203.4565v4 (2012)
- [56] Bruno, N., Martin, A., Sekatski, P., Sangouard, N., Thew, R., and Gisin, N.: Displacing entanglement back and forth between the micro and macro domains. arXiv:quant-ph/1212.3710 (2012)
- [57] Vandersypen, L.M.K., Steffen, M., Breyta, G., Yannoni, C.S., Cleve, R., Chuang, I.L.: Implementation of a three-quantum-bit search algorithm. arXiv:quant-ph/9910075v2 (2000)

- [58] Steffen, M., van Dam, W., Hogg, T., Breyta, G., Chuang, I.: Experimental implementation of an adiabatic quantum optimization algorithm. arXiv:quant-ph/0302057v2 (2003)
- [59] Ji, Y., Chung, Y., Sprinzak, D. , Heiblum, M., Mahalu, D. , Shtrikman, H.: An Electronic Mach-Zehnder Interferometer. *Nature* **422**, 415 (2003)
- [60] Kang, K. : Electronic Mach-Zehnder quantum eraser. *Phys. Rev. B* **75**, 125326 (2007).
- [61] Kang, K., Lee, K.H. : Violation of Bells inequality in electronic Mach-Zehnder interferometers. *Physica E* **40**, 1395 (2008)
- [62] Dressel,J., Choi,Y., Jordan,A. N.: Measuring which-path information with coupled electronic Mach-Zehnder interferometers. *Phys. Rev. B* **85**, 045320 (2012).
- [63] Buscemi, F., Bordone, P. and Bertoni, A. Electron interference and entanglement in coupled 1D systems with noise. *The European Physical Journal D* **66**, 111 (2012).
- [64] Weisz, E., Choi, H. K., Sivan, I., Heiblum, M., Gefen, Y., Mahalu, D., Umansky, V.: An Electronic Quantum Eraser arXiv:1309.2007v2 [cond-mat.mes-hall] (2013)
- [65] Mermin,N.D.: What's Wrong with this Pillow? *Physics Today*. **42**, 4 (1989) <http://dx.doi.org/10.1063/Y1.2810963> (Mermin's earlier characterization of the Copenhagen interpretation; sometimes attributed to Feynman)